

# Theory Canada 9

## June 12–15, 2014

### Wilfrid Laurier University



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# ABSTRACTS

## QUANTUM INFORMATION

### **INVITED TALK: Extending the quantum domain with quantum optical systems**

Christoph Simon, *Institute for Quantum Science and Technology, University of Calgary*

Quantum optical systems are particularly well suited to pushing the boundaries of the domain where basic quantum effects such as superposition and entanglement have been observed. I will focus on two examples. First, I will describe a proposal to create entanglement over global distances by combining satellite-based quantum communication links and so-called quantum repeaters. I will highlight the need for quantum non-demolition measurements of photonic qubits in this context and describe an idea for realizing such measurements. Second, I will discuss recent proposals and experiments on the creation and detection of entangled states involving macroscopic numbers of photons (experimental) and phonons (theoretical for now). A common element playing a key role in both examples is the storage of quantum states of light in material systems.

### **Causal models for the quantum world**

Katja Ried and Robert W. Spekkens, *Perimeter Institute for Theoretical Physics*

Science aims to achieve more than merely observing correlations between events: it seeks to explain these correlations in terms of causal influences. A rigorous definition of causation and a comprehensive framework of causal models have been developed for the analysis of classical variables [1, 2]. In problems ranging from machine learning to epidemiology, causal models proved themselves an accurate representation of “how a system works” – in particular, they tell us something about how information flows from one event to another. Can we provide a similar account of the relations between a set of quantum variables? I will discuss ways in which classical causal models must be adapted to accommodate quantum variables, highlighting how causation and information processing are different from the classical case.

One major task in the field of causal modeling is causal inference: determining the causal relations between a set of variables, given only observational data. For instance, if one observes correlations between two variables, does this show a direct causal influence, or is it due to some hidden common cause? In the classical world, this particular problem is impossible to solve: any pattern of correlations one may observe between two variables is compatible both with a direct causal influence of one on the other, and with a common cause influencing both. For quantum variables, on the other hand, we show that the correlations do encode a signature of the causal structure, allowing one to solve the causal inference problem. We illustrate this with data from a proof-of-concept experiment that corroborates our scheme [3].

[1] J. Pearl, *Causality: reasoning, models and inference*. Cambridge University Press, 2009.

[2] P. Spirtes, C. Glymour and R. Scheines, *Causation, prediction and search*. MIT Press, 2000.

[3] M. Agnew, L. Vermeyden, K. Ried, R. W. Spekkens and K. Resch, *Unified tomography of states and processes*, unpublished.

### **Weak measurements with operational constraints**

Aharon Brodutch, *University of Waterloo*

Weak values are a useful characteristic of quantum systems with past (pre selected) and future (post selected) boundary conditions. Operationally they are effective interaction coefficients for weak measurements. The standard weak measurement scheme, however, cannot be used to measure all weak values due to operational constraints. These range from conceptual constraints such as locality [1, 2] to implementation constraints such as the inability to perform projective measurements in ensemble systems[3].

In this talk I will outline a new method for directly coupling to a wide range of weak values that correspond to observables that cannot be observed using the von Neumann scheme. As an example I will introduce the four path paradox - a variant of the three box paradox [4] and show how it could be implemented as a quantum algorithm. Unlike the three box paradox the four path paradox involves measurements of sequential operators that cannot be measured in the standard scheme. The result boosts the significance of weak values related to non-local and sequential observables.

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- [2] A. Brodutch and L. Vaidman, in Journal of Physics: Conference Series (2009), vol. 174, p. 012004, URL <http://iopscience.iop.org/1742-6596/174/1/012004>.
- [3] D. Lu, A. Brodutch, J. Li, H. Li, and R. Laflamme, New Journal of Physics in press (2014), arXiv:1311.5890.
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## INVITED TALK: Delegating Private Quantum Computations

Anne Broadbent, *University of Ottawa*

Given the technological challenge in building quantum computers, it is likely that their initial availability will be in a client-server configuration. We address the question of privacy in this scenario, by showing that an almost-classical client can delegate the execution of any quantum computation, where the data uploaded to the server is encrypted via the one-time pad. In order to do this, the quantum power required of the client is limited to being able to prepare random BB84 states. We give a simulation-based security definition and a rigorous proof of security using a transformation to an entanglement-based protocol.

## Communication Complexity of One-Shot Remote State Preparation

Shima Bab Hadiashar<sup>1</sup>, Matthias Christandl<sup>2</sup>, Ashwin Nayak<sup>1</sup>, and Renato Renner<sup>3</sup>

<sup>1</sup>University of Waterloo, <sup>2</sup>University of Copenhagen, <sup>3</sup>ETH Zurich

Remote state preparation (RSP) is a variant of quantum teleportation introduced by Lo [4] in which Alice knows a classical description of the quantum state and wishes to help Bob prepare it, with the aid of shared entanglement and classical communication. In particular, Alice and Bob share entangled qubits, Alice is given the description of a state  $Q(x)$ , chosen from a subset of quantum states  $\{Q(1), \dots, Q(n)\}$ , and their goal is to prepare that quantum state on Bob's side using an LOCC (Local Operations and Classical Communication) protocol. A relaxed version of this task is *approximate remote state preparation* in which the goal is to prepare an approximation  $\sigma_x$  of a quantum state  $Q(x)$ . We define the error of a protocol for approximate remote state preparation in terms of the fidelity between  $Q(x)$  and  $\sigma_x$ . We say a protocol prepares a state with *worst-case error* at most  $\epsilon$ , if for every  $x \in \{1, \dots, n\}$ ,  $F(Q(x), \sigma_x) \geq \sqrt{1 - \epsilon^2}$ . Similarly, a protocol has *average-case error* at most  $\epsilon$  with respect to a probability distribution  $p$ , if  $\sum_{x=1}^n p_x F(Q(x), \sigma_x) \geq \sqrt{1 - \epsilon^2}$ .

In [4], Lo gave several examples of ensembles which can be remotely prepared using a one-way protocol with classical communication cost less than quantum teleportation. However, he conjectured that to prepare an arbitrary pure state remotely, one needs to send classical bits in the same asymptotic rate as in quantum teleportation i.e., 2 classical bits per qubit. In [1], it was shown that the asymptotic classical communication cost of remote state preparation of a general state ranges from one bit per qubit in the high entanglement limit to infinite bit per qubit in the case of no shared entanglement. Later in [2], Bennett *et al.* showed that approximate remote state preparation with small worst-case error  $\epsilon$  requires asymptotic rate of one bit of classical communication per qubit from Alice to Bob. They also showed that this amount of classical communication is sufficient. In [3], Jain studied remote state preparation in the *one-shot* scenario. He considered the total communication cost instead of the rate of communication in the case that there is no restriction on the amount of entanglement. He showed that the communication cost required for *exact* remote state preparation is at least  $T(Q)/2$  and approximate RSP with worst-case error at most  $\epsilon$  can be solved with communication at most  $\frac{8}{(1 - \sqrt{1 - \epsilon^2})^2} (4T(Q) + 7)$ , where  $T(Q)$  denotes

maximum possible mutual information in an encoding  $Q$ . It is rather unsatisfactory that the upper bound for the approximation version be *larger* than the lower bound for the *exact* version of the problem. We would expect the complexity of the problem to decrease as the error in approximation increases.

In this work, we give a tighter characterization of the communication complexity of *one-shot* remote state preparation in two different cases. First, we consider RSP with average-case error at most  $\epsilon$ , and show that its communication complexity is bounded from below and above by the notion of smooth max-information Bob has about Alice's input. Then, we consider RSP with worst-case error at most  $\epsilon$ , and give lower and upper bounds for its communication complexity in terms of smooth max-relative entropy and show that our upper bound is a factor of  $\log N$  tighter than that of [3]. Also, we show that the communication cost may reduce dramatically by allowing more error and considering average-case error instead of worst-case error. In particular, we show that for every  $\epsilon \in [0, \frac{1}{\sqrt{2}})$ , there exist a set of quantum states of dimension  $N$  and a probability distribution  $p_\epsilon$  over this set for which there is a  $\log N$  gap between the worst-case error and average-case error remote preparation of that set. In addition, we exhibit a set of quantum states, which illustrates the heavy dependence of the the gap between the worst-case error and average-case error communication complexity on the error parameter  $\epsilon$ . This example also shows that the more skewed the probability distribution is, the bigger the gap between worst-case and average-case error can be.



average-case error communication complexity on the error parameter  $\epsilon$ . This example also shows that the more skewed the probability distribution is, the bigger the gap between worst-case and average-case error can be.

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## PARTICLES AND FIELDS

### INVITED TALK: Spin and Long-Range Forces: The Unfinished Tale of the Last Massless Particle

Natalia Toro, *Perimeter Institute for Theoretical Physics*

The success of gauge theory descriptions of Nature follows simply, in hindsight, from Lorentz symmetry, quantum mechanics, and the existence of interacting massless particles with spin. Yet, remarkably, the most generic type of massless particle spin has never been seriously examined: Wigner's so-called "continuous spin" particles (CSPs), which have a tower of polarization states carrying all integer or half-integer helicities that mix under boosts. I will introduce the basic physics of this type of particle, present recent progress in understanding them using both S-matrix arguments and field theory, and explain how they might be relevant to Nature.

### The application of light front holographic QCD to B physics

Muhammad Ahmady, *Mount Allison University*

Holographic AdS/QCD is a model put forward by Brodsky and de Teramond which deals with the hadronic bound states in a relativistic invariant fashion. We have used the light front wavefunction obtained from AdS/QCD to calculate the distribution amplitudes of the light vector mesons like  $\rho$  and  $K^*$ . As a result, we are able to calculate  $\Lambda_{QCD}/m_b$  effects in  $B_{(s)} \rightarrow (\rho, K^*)\gamma$  and the form factors for  $B \rightarrow \rho, k^*$  transitions. I present some of our results and predictions as compared to the most recent experimental data.

### Exploring the Spectrum of Heavy Quarkonium Hybrids with QCD Sum Rules

Robin Kleiv, *University of the Fraser Valley*

In recent years, many unanticipated hadrons have been discovered by the Babar, Belle, CLEO, D0 and LHCb experiments. These hadrons exist among conventional heavy quarkonium mesons (i.e. charmonia and bottomonia) and are referred to as heavy quarkonium-like or XYZ states. Because many of the XYZ states are difficult to interpret as charmonium or bottomonium mesons, they are widely considered to be exotic hadrons. Hybrid mesons, which are mesons that include explicit gluonic degrees of freedom, are one such possibility. The masses of heavy quarkonium hybrids can be predicted using the techniques of QCD sum rules, helping to ascertain the identities of the XYZ states. In this talk, I will provide a brief overview of the XYZ states, exotic hadrons and QCD sum rules. I will also discuss our recent work in which heavy quarkonium hybrid masses were determined for a large number of distinct JPC channels using QCD sum rules. Our results will be compared to those of other theoretical approaches, such as lattice QCD. I will also discuss the implications of our work for the XYZ states.

### INVITED TALK: Dark Forces, Dark Matter, and the GeV-Scale Discovery Frontier

Philip Schuster, *Perimeter Institute for Theoretical Physics*

The search for dark matter and new forces mediated by sub-GeV particles with very weak coupling to matter ("dark forces") is an emerging frontier in fundamental physics. I will present the theoretical motivations for dark forces and their possible connections to dark matter and the anomalous magnetic moment of the muon. I will also discuss strategies, results, and prospects for searches at high-energy colliders, flavor factories, and dedicated fixed-target experiments.

# Charge Quantization from a Number Operator

Cohl Furey

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**Background.** It is known that electric charge quantization arises in the presence of Dirac monopoles, and in the context of grand unified theories. More recently, Hellerman, Kehayias, and Yanagida have proposed that charge quantization can come about in non-linear sigma models, [1].

Here, we reveal a striking new resolution to the question, *Why is electric charge quantized?*. Perhaps electric charge is quantized because it happens to be given by a number operator.

To this end, we consider the possibility of a more abstract mathematical structure behind the Lie group representations of the standard model. One option for this structure is that of the normed division algebras over  $\mathbb{R}$ . Only four of these algebras exist, and three of them, namely the reals, the complex numbers and the quaternions (sigma matrices), are already central to particle physics. Here, we explore the role of the fourth division algebra, the octonions.

In earlier years, [2], Günaydin and Gürsey showed  $SU_c(3)$  quark structure in the split octonions. Later, in [3], they showed anti-commuting ladder operators,  $\alpha_i$ , within that model. Our new results are consistent with the chromodynamic quark model of [2] and [3].

**New results.** We show a direct route, via ladder operators, to a new  $U(1)$  generator. This  $U(1)$  generator behaves like electric charge, thereby allowing us to identify not only the quark states, but also new states behaving like the electron and neutrino. Our proposed electric charge turns out to be proportional to a number operator, consequently illuminating why it is quantized.

$$Q \equiv \frac{1}{3} \sum_{i=1}^3 \alpha_i^\dagger \alpha_i = \frac{N}{3}. \quad (1)$$

Using only a trio of ladder operators,  $\alpha_i$ , and their conjugates, we construct a pair of *minimal left ideals*, which is shown to transform under  $SU_c(3)$  and  $U_{em}(1)$  as does a full generation of the standard model.

The first of the minimal left ideals,  $S^u$ , behaves under electric charge,  $Q$ , and  $su_c(3)$ ,  $\Lambda$ , according to the

following table.

$\underline{Q}$	$\underline{\Lambda}$	$\underline{S^u}$	$\underline{\text{ID}}$
0	1	$\omega\omega^\dagger$	$\nu$
1/3	$\bar{3}$	$\alpha_i^\dagger\omega\omega^\dagger$	$\bar{d}_i$
2/3	3	$\alpha_i^\dagger\alpha_j^\dagger\omega\omega^\dagger$	$u_k$
1	1	$\omega^\dagger$	$e^+$

(2)

Here, objects  $\omega\omega^\dagger$ ,  $\alpha_i^\dagger\omega\omega^\dagger$ ,  $\dots$  are simply basis vectors of  $S^u$ . The object  $\omega$  is defined as  $\omega \equiv \alpha_1\alpha_2\alpha_3$ .

The complex conjugate of  $S^u$  gives a new linearly independent ideal,  $S^d$ , in the space of octonionic maps. In this new formalism, each particle and anti-particle pair are related by *only* the complex conjugate,  $i \mapsto -i$ . These two minimal left ideals, or generalized spinors,  $S^u$  and  $S^d$ , are depicted in the figure below.

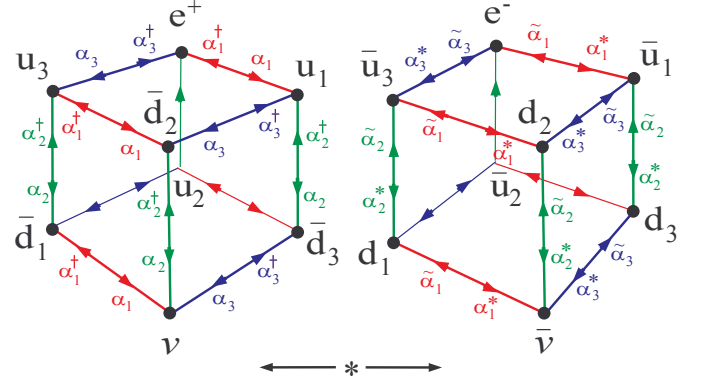


FIG. 1. A full generation represented by cubes  $S^u$  (left) and  $S^d$  (right). Quark and electron states may be viewed as excitations from the neutrino or anti-neutrino.

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  - [2] M. Günaydin, F. Gürsey, J. Math. Phys., Vol. 14, No. 11 (1973)
  - [3] M. Günaydin, F. Gürsey, PRD, Vol. 9, No. 12 (1974)

## THEORETICAL CONDENSED MATTER AND COMPUTATIONAL PHYSICS

### INVITED TALK: Semiconductor and Graphene Quantum Dots

Paweł Hawrylak, *National Research Council of Canada*

We describe here recent theoretical and experimental results on both lateral gated and self-assembled semiconductor quantum dots and on graphene quantum dots. Using LCHO-CI, extended Hubbard, CI and DMRG methods we describe lateral quantum dot molecules with controlled electron numbers in each dot and discuss potential use of such molecules as building blocks of a field effect transistor with a macroscopic quantum state based on artificial Haldane gap material, of a coded qubit based on chirality, GHZ maximally entangled state and Berry's phase generators[1]. We also describe topological phases driven solely by e-e interactions in a quadruple quantum dot molecule[2] and by spin orbit interaction in InAs [3] and HgTe based quantum dots[4]. We next turn to CdTe quantum dots containing single magnetic ions and discuss quantum interference and Kondo-like effects in fine structure of Mn ions interacting with excitons[5] and bi-excitons[6]. Finally, we describe one atom thick semiconductor quantum dots made of graphene and compare them with semiconductor quantum dots. Using a combination of DFT, tb, HF, CI and GW-BSE approaches we show that their electronic, optical and magnetic properties can be engineered by the size, shape, type of edge, topology and number of layers [7-12]. We focus on their optical and magnetic properties, and their control with external gate, carrier density, electric field and light. A possibility of realizing a fully integrated carbon-only quantum circuit will be discussed.

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- [12] I. Ozfidan, M. Korkusinski, A.D. Güçlü, J. McGuire, P. Hawrylak, Phys. Rev. B **89**, 085310 (2014).

### INVITED TALK: Classical and Quantum Spin State in Frustrated Magnetic Materials

Michel Gingras *University of Waterloo, and Canadian Institute for Advanced Research*

A frustrated system is one whose symmetry precludes the possibility that every pairwise interaction ("bond") in the system can be satisfied at the same time. Such systems are common in all areas of physical and biological science. In the most extreme cases, they can have a disordered ground state with "macroscopic" degeneracy; that is, one that comprises a huge number of equivalent states of the same energy. Pauling's description of the low-temperature proton disorder in water ice was perhaps the first recognition of this phenomenon and remains the paradigm. In recent years, a new class of magnetic substance has been characterized, in which the disorder of the magnetic moments at low temperatures is precisely analogous to the proton disorder in water ice. These substances, known as spin ice materials<sup>1</sup>, are perhaps the "cleanest" examples of such highly frustrated systems yet discovered. They offer an unparalleled opportunity for the study of frustration in magnetic systems at both an experimental and a theoretical level.

Remarkably, spin ices provide one of very few experimentally realized examples of fractionalization because their elementary excitations can be regarded as magnetic monopoles and, over some temperature range, the spin ice materials are best described as liquids of these emergent charges. In the presence of quantum fluctuations, one can obtain, in principle, a quantum spin liquid descended from the classical spin ice state characterized by emergent photon-like excitations. Whereas in classical spin ices the excitations are akin to electrostatic charges, in the quantum spin liquid these charges interact through a dynamic and emergent electromagnetic field. This is the so-called quantum spin ice state<sup>2</sup>.

In this talk, I will describe the essential physics of spin ice, as it is currently understood and mention the latest development in the study of such quantum spin ices and the experimental search for this state of matter among real materials.

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### **INVITED TALK: Chasing the Magnetic Butterflies in Nanoscale Fields**

Tapash Chakraborty, *University of Manitoba*

Two-dimensional electrons simultaneously exposed to a perpendicular magnetic field and a two-dimensional periodic potential exhibit an intricate energy spectrum with the fractal pattern that resembles a butterfly. Over the years, there have been many attempts to observe this unique pattern but with limited success. A breakthrough has now been achieved finally in monolayer and bilayer graphene. The chapter, however, is not yet closed and there remains many questions that need to be answered.

### **Geometric spin manipulation in semiconductor quantum dots**

Sanjay Prabhakar,<sup>1</sup> Roderick Melnik,<sup>1</sup> and Akira Inomata<sup>2</sup>

<sup>1</sup>*M2NeT Laboratory, Wilfrid Laurier University, 75 University Avenue West, Waterloo, Ontario N2L 3C5, Canada*

<sup>2</sup>*Department of Physics, State University of New York at Albany, Albany, New York 12222, USA*

We propose a method to flip the spin completely by an adiabatic transport of quantum dots. We show that it is possible to flip the spin by inducing a geometric phase on the spin state of a quantum dot. We estimate the geometric spin flip time (approximately 2 ps) which turned out to be much shorter than the experimentally reported decoherence time (approximately 100 ns) that would provide an alternative means of flipping the spin before reaching decoherence. It is important that both the Rashba coupling and the Dresselhaus coupling are present for inducing a phase necessary for spin flip. If one of them is absent, the induced phase is trivial and irrelevant for spin-flip (*Applied Physics Letters* 104, 142411, 2014).

### **Floquet-Bloch Waves Photonic Crystals with Absorption**

G. V. Morozov<sup>1\*</sup> and D. W. L. Sprung<sup>2</sup>

<sup>1</sup>*Scottish Universities Physics Alliance (SUPA), Thin Film Centre, University of the West of Scotland, Paisley PA1 2BE, Scotland, United Kingdom*

<sup>2</sup>*Department of Physics and Astronomy, McMaster University, Hamilton, ON L8S 4M1, Canada*

We consider light propagation in one-dimensional photonic crystals with absorption. The solutions of the wave equation are represented in terms of two Floquet-Bloch waves. In case of absorption, the Floquet multipliers and the Bloch wave number associated with those waves become complex for all frequencies of the propagating light, which makes difficult to separate the frequency spectrum into allowed bands and bandgaps. To address the issue, we investigate the dependence of the Floquet multipliers on frequency (wavenumber) in a complex plane. Then, the relation between the Bloch wave number and the multipliers for the absorptive case is established. As a consequence, this relation is also clarified for the bandgaps of lossless (non-absorptive) crystals. The obtained results should facilitate the proper parameterization of the transfer matrix for the absorptive crystals as well as for the bandgaps of lossless crystals.

- [1] G. V. Morozov and D. W. L. Sprung, "Floquet-Bloch waves in one-dimensional photonic crystals," *Europhys. Lett (EPL)*, vol. 96, 54005, Dec. 2011.

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### **Modeling of coarse-grained structures for the RNA nanotube using molecular dynamics simulation**

Shyam Badu<sup>1</sup>, Roderick Melnik<sup>1</sup>, Maxim V Paliy<sup>2</sup>, Sanjay Prabhakar<sup>1</sup> and Bruce A Shapiro<sup>3</sup>

<sup>1</sup>*M2Net lab, Wilfrid Laurier University, Waterloo, ON N2L 3C5,*

<sup>2</sup>*Department of Chemistry, University of Western Ontario, London, ON,*

<sup>3</sup>*Center for Cancer Research Nanobiology Program, Frederick, MD 21702, USA*

The self assembly of the RNA nanostructures is straightforward due to high flexibility of RNA molecules. We construct the novel RNA nanoclusters- the RNA nanotubes made of several nanorings. We study the structural properties (i.e., the Root Mean Square Deviation (RMSD), the ra-dius of gyration and radial distribution function (RDF)) of such RNA nanotubes up to the size of about 20nm in physiological solutions that can be used for drug delivery into the human body. We model RNA nanotubes by utilizing the molecular dynamics simulation method implemented in NAMD and VMD. Furthermore, we calculate the histograms for the bond angles and the dihedral

angles. From dihedral angle histogram we see the characteristics of the links used to built the nan- otube. Using these all-atom MD simulations results we develop the coarse-grained models which leads to a possibility of explaining the properties of complex biological systems at large scales. molecules. For the coarse-grained structures we have performed the Molecular dynamics simulation using DL-POLY to calculate the radial distribution function. We also present the root mean square deviation for the system of the coarse-grained model to demonstrate the equilibration of the system. The change in these features with the size of the nanotube will be discussed.



## GRAVITATION AND COSMOLOGY

### INVITED TALK: Shape Dynamics

Tim Koslowski, *Department of Mathematics and Statistics University of New Brunswick, Fredericton, NB*

Classical gravity can be constructed from relational first principles. These principles lead to a description of gravity in terms of the evolution of spatial conformal geometry. This “shape dynamics” description of gravity is locally indistinguishable from classical General Relativity. In this talk, I will introduce the shape dynamics description of classical gravity and explain why it is almost indistinguishable from General Relativity. I then assume the shape dynamics description of gravity to be the fundamental description of gravity. I then explain how the standard description of gravity in terms of spacetime geometry emerges from shape dynamics. I will conclude with the describing the present status of quantum shape dynamics and explain the differences with quantum General Relativity.

### INVITED TALK: Non-Commutative Geometry, Non-Associative Geometry and the Standard Model of Particle Physics

Latham Boyle (in collaboration with Shane Farnsworth), *Perimeter Institute*

Connes has developed a notion of non-commutative geometry (NCG) that generalizes Riemannian geometry, and provides a framework in which the standard model of particle physics, coupled to Einstein gravity, may be concisely and elegantly reformulated. We point out that his formalism may be recast in a way that generalizes immediately from non-commutative to non-associative geometry. In the process, several of the standard axioms and formulae are conceptually reinterpreted. This reformulation also suggests a new constraint on the finite NCG corresponding to the standard model of particle physics. Remarkably, this new condition resolves a long-standing puzzle about the NCG embedding of the standard model, by eliminating from the action a collection of 7 unwanted terms that previously had to be removed by assumption.

### Horizon thermodynamics and spacetime mappings

Valerio Faraoni and Vincenzo Vitagliano, *Bishops University*

Apparent/trapping horizons replace the teleological concept of event horizon when black holes are dynamical. Conformal transformations and generalized Kerr-Schild transformations are often employed to generate and study dynamical black holes. We study the behaviour under such transformations of thermodynamics quantities for apparent horizons, including the Misner-Sharp-Hernandez mass (internal energy), the Kodama vector, surface gravity, and temperature. These quantities transform differently from what would be expected according to naive arguments.

Based on V. Faraoni & V. Vitagliano, *Phys. Rev. D* 89, 064015 (2014)

### Negative mass configurations in de Sitter space

S. Mbarek and M. B. Paranjape, *Université de Montreal*

We consider matter represented by a perfect fluid and we show the existence of configurations which everywhere satisfy the dominant energy condition, the condition of hydrostatic equilibrium and give rise to geometries that asymptotically approach the exact solution of a negative mass Schwarzschild-de Sitter space-time. Such configurations could have arisen in the early universe during the inflationary epoch.

### The Echo of the Early Universe

Eduardo Martin-Martinez, *Institute for Quantum Computing and Perimeter Institute*

By applying quantum informational and optical tools to quantum gravity theories in the very early universe, we show that the fluctuations of quantum fields as seen by late comoving observers are significantly influenced by the history of the early universe, transmitting information about the nature of the universe in timescales when quantum gravitational effects were non-negligible. This might be observable even nowadays thus used to build falsifiability tests of quantum gravity theories.

## MATHEMATICAL PHYSICS

### INVITED TALK: Chern-Simons Gauge Theory and Symplectic Quantum Mechanics

Lisa Jeffrey, *University of Toronto*

I will describe the relation between Chern-Simons gauge theory (where the fields are connections on 3-manifolds and the action is the Chern-Simons functional and another gauge theory called symplectic quantum mechanics (where the fields are paths in a symplectic manifold satisfying prescribed boundary conditions and the action is the symplectic action functional). This comparison is most useful when the three-manifold is a mapping torus of a 2-manifold equipped with a symplectic diffeomorphism.

### Dynamical Equations Of Periodic Systems Under Constant External Stress

Gang Liu, *High Performance Computing Virtual Laboratory Queen's University*

Periodic boundary conditions are widely used in the simulation of systems with an extremely large number of particles, and the period vectors become a degree of freedom. In this work, dynamical equations for the periods are generated by applying Newtonian Dynamics onto halves of the system while considering constant external stress explicitly. Then statistics over system translations and directions of particle movements are applied to the dynamical equations. This leads to the full interaction and kinetic-energy terms in the internal stress. In the resulting expressions, the periods are driven by the imbalance between the internal and external stresses. (The manuscript has been published as <http://arxiv.org/pdf/cond-mat/0209372v10.pdf>, and submitted to the Canadian Journal of Physics for formal publication on Jan. 08, 2014.)

### The supersymmetric Ruijsenaars-schneider model and its integrability

O. Blondeau-Fournier, *Université Laval*

The Calogero-Sutherland (CS) model describes a quantum mechanical system of  $N$  particles interacting pairwise in 1 dimension. Its one-parameter deformation, called the (trigonometric) Ruijsenaars-Schneider (RS) model, is (often) considered as its relativistic extension since the algebra satisfied by its hamiltonian, momentum and boost operators, is the Poincaré algebra. Moreover, in the limit  $c \rightarrow \infty$  (where  $c$  is playing the role of speed of light), it reduces to the CS model. These two models are integrable because there are  $2N$  mutually commuting quantities whose set includes the hamiltonian. It is also a remarkable result that the eigenfunctions of these models are the Jack and Macdonald symmetric polynomials (in the CS and RS cases respectively).

Supersymmetric generalization of quantum mechanical models which preserves integrability is an important field of research in mathematical physics. For the case of CS model, this has been formulated in 1993. Its eigenfunctions (obtained in early 2000) are superpolynomials that depend upon two families of variables – commuting and anticommuting – and which remain invariant when both type of variables are simultaneously permuted. These are naturally called the Jack superpolynomials. Furthermore, it turns out that they can also be defined purely by combinatorial methods and their study opened up the field of symmetric functions in superspace.

On the other hand, the quest for an integrable one-parameter deformation of the supersymmetric CS model is a long standing problem. The difficulty lies in that standard supersymmetrization methods no longer work in case of the RS model. Very recently, such an extension has been discovered and its eigenfunctions are the superspace analogue of Macdonald polynomials which have been constructed in the last two years by combinatorial techniques. The proof of integrability of this model makes use of the Hecke algebra and the related Cherednik operators.

In this talk, I will present the supersymmetric formulation of the trigonometric RS model. Its underlying algebra, that is, the (anti)commutation relations satisfied by the hamiltonian, momentum, boost and supercharges, is now the Poincaré superalgebra. Then, I will present its eigenfunctions, the Macdonald superpolynomials, with some of their (amazing) combinatorial properties. Finally, I will show how the commuting conserved quantities can be constructed with the aid of the Cherednik operators. These original results are based on the following references.

- [1] O. Blondeau-Fournier, P. Desrosiers, L. Lapointe and P. Mathieu, Macdonald polynomials in superspace: conjectural definition and positivity conjectures, *Lett. Math. Phys.* 101 (2012) 27.
- [2] O. Blondeau-Fournier, P. Desrosiers, L. Lapointe and P. Mathieu, Macdonald polynomials in superspace as eigenfunctions of commuting operators, *J. of Comb.* 3 (2012) 495.
- [3] O. Blondeau-Fournier, P. Desrosiers and P. Mathieu, The supersymmetric Ruijsenaars-Schneider model, *arXiv:1403.4667* (2014)

## **INVITED TALK: The rotation group Bivariate Krawtchouk polynomials and state transfer on a 2-d lattice**

Luc Vinet, *CRM Université de Montréal*

I shall explain how polynomials in two discrete variables that are orthogonal with respect to the trinomial distribution arise as matrix elements of the reducible representation of the rotation group  $SO(3)$  on the space of the state vectors of the 3-d harmonic oscillator with fixed energy. This algebraic interpretation of the bivariate Krawtchouk polynomials allows as I shall illustrate, for a straightforward characterization of these functions. As an example of their applications, I shall review their use in the design of a spin lattice with remarkable state transfer properties. Based on work with V. Genest (Montreal), H. Miki (Kyoto) and A. Zhedanov (Donetsk).

## **Large parameter limits of conformal blocks**

Connor Behan, *Department of Physics and Astronomy, University of British Columbia*

A surprising fact is that conformally invariant quantum field theories can be “bootstrapped”. This is because they can be defined without reference to a Lagrangian or Hamiltonian. All one needs are the spins and scaling dimensions of the local operators and the three-point function coefficients. Some choices for these numbers correspond to the critical Ising model or the supersymmetric Yang Mills model, but most are unphysical and must be ruled out numerically. I will present new results about conformal blocks - a family of functions that are important in these numerical studies. One result is that the blocks approach monomials as the dimension difference between two operators becomes large. Another is that they lose dependence on dimension differences as the spacetime dimension becomes large. Most progress with the conformal bootstrap has been made by studying the correlation functions of scalar operators. Studies of higher spin operators are expected to lead to a wealth of information particularly in the non-supersymmetric case. The analogous functions that would be needed, called spinning conformal blocks, are much further from being well understood. I will touch on large parameter limits of these functions as well and what they can tell us about the road ahead.

## STRING THEORY, QUANTUM GRAVITY

### INVITED TALK: Matter matters in asymptotically safe quantum gravity

Astrid Eichorn, *Perimeter Institute for Theoretical Physics*

Abstract: I will discuss the asymptotic safety scenario, which provides a way to construct a quantum field theory for gravity. This scenario relies on an interacting fixed point of the Renormalization Group flow, generalizing the idea of asymptotic freedom, which is well-known from the Standard Model of particle physics. I will focus on the impact of dynamical matter degrees of freedom on the scenario. In particular, I will discuss whether future observations of new particles, e.g. at the LHC or future colliders, could be used to rule out this quantum gravity model.

### INVITED TALK: On Spacetime Entanglement

Rob Myers, *Perimeter Institute for Theoretical Physics*

I review a recent conjecture that the Bekenstein-Hawking formula,  $S=A/4G$ , describes the entanglement entropy for general regions of spacetime in a theory of quantum gravity. I also discuss how this result is realized in the context of the AdS/CFT correspondence.

### Near Horizon Geometries and Black Hole Thermodynamics: a Lesson in Limits

Sean Stotyn, *University of British Columbia*

It is well known that the spacetime in the vicinity of an extremal black hole horizon has an enhanced symmetry group. This enhancement of symmetry is due to the attractor mechanism, whereby the entropy of extremal black holes cannot depend on any moduli of the theory. The near horizon geometry (NHG) in  $D$  dimensions is typically a product (either direct or twisted) of  $AdS_2$  and  $SD-2$ , which has allowed for an interpretation of extremal black hole entropy in terms of microscopic states in a dual CFT. However, the standard limiting procedure for finding the NHG obscures the connection between the NHG and the bulk space-time: in particular, it is not immediately clear what the horizon maps to in the NHG and which Killing symmetries in the NHG are present in the bulk spacetime and which are not. The standard limiting procedure further obscures the meaning of the NHG: I will show that the NHG is to be thought of as a tangent space to the horizon, with the degenerate horizon mapping to a degenerate null hypersurface in the NHG. This distinction becomes extremely important when considering the extremal limit of Schwarzschild de Sitter. Standard lore would have one believe that the volume between the coalescing horizons remains finite in the extremal limit, leading to the Nariai spacetime whose two cosmological horizons are interpreted as the two limiting horizons. Importantly, this suggests that extremal Schwarzschild de Sitter is at finite temperature. However, I will show that this interpretation is problematic; the Nariai spacetime should really be thought of as the NHG of the extremal Schwarzschild de Sitter spacetime. When viewed this way, I will show that extremal Schwarzschild de Sitter is indeed at zero temperature, as an extremal horizon should be.

### Black hole chemistry

David Kubizňák, *Perimeter Institute for Theoretical Physics*

The mass of a black hole has traditionally been identified with its energy. We describe a new perspective on black hole thermodynamics, one that identifies the mass of a black hole with chemical enthalpy, and the cosmological constant as thermodynamic pressure. This leads to an understanding of black holes from the viewpoint of chemistry, in terms of concepts such as Van der Waals fluids, reentrant phase transitions, and triple points. Both charged and rotating black holes exhibit novel chemical-type phase behaviour, hitherto unseen.

# Gravity from Order and Number: Causal Sets

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*Waterloo, ON N2L 2Y5*

## *Abstract*

Among the various ideas put forward in the search for a theory of quantum gravity, the causal set hypothesis is distinguished by its logical simplicity and by the fact that it incorporates the assumption of an underlying spacetime discreteness organically and from the very beginning. After presenting the problem of quantum gravity in general, I will precis the causal set programme and touch on some old and some recent developments.

## POSTER SESSION

### The self-force in five-dimensions

Matthew J. S. Beach, Eric Poisson, Bernhard G. Nickel, *University of Guelph*

There is currently a lack of intuition regarding the electromagnetic self-force experienced by a charged particle when it is placed near a black hole. To further probe the nature of this force, we generalize the self-force calculation to a static five-dimensional spherically symmetric spacetime and attempt to relate it to an analogue model. By decomposing the electrostatic potential into a sum of higher-dimensional spherical harmonics and performing a regularization method based on a Hadamard Green's function, we are able to obtain the self-force as a convergent mode-sum. Our calculation was implemented in three different ways. Firstly, we used a numerical algorithm in Maple. Secondly, we expressed the self-force as an series expansion about infinity, and finally, we derived a closed-form solution from the power series. In contrast to the four-dimensional model, we find the self-force to depend on a regularization parameter  $s$  which can be interpreted as the radius of the particle. At large distances, the self-force is repulsive and is related to an elementary electrostatic model, however, as the distance between the event horizon and the particle decreases, the force becomes attractive and the model fails.

### Quantum Collect-Calling

Robert H. Jonsson, *University of Waterloo*

We show that it is possible to use a massless field in the vacuum to communicate in such a way that the signal travels slower than the speed of light and such that no energy is transmitted from the sender to the receiver. Instead, the receiver has to supply a signal-dependent amount of work to switch his detector on and off. This type of signalling is related to Casimir-like interactions and it is made possible by dimension ---and curvature--- dependent subtleties of Huygens' principle [1].

To model the receiver and sender we use Unruh-DeWitt particle detectors (UDW). Since its introduction, the UDW detector has proven to be an important tool for investigating QFT on curved spacetimes. More recently, in the field of Relativistic Quantum Information, the behaviour of UDW detectors has been studied from a Quantum Information theoretical perspective [2].

The results presented in this talk are part of a research program studying communication between UDW detectors [3-5]: Understanding the effect of curvature and relativistic motion on our capability to communicate via quantum fields, measured in terms of the channel capacity, could provide us with a framework to quantify the causal structure of spacetime.

The effects we discuss in this talk originate from the behaviour of the field commutator. On spacetimes that do not obey the strong Huygens' principle, the commutator is non-vanishing in the future lightcone. This is, in general, the case in curved spacetimes but in particular it is true in two- and three-dimensional Minkowski space.

It is interesting to see that the signals we are describing appear in leading order of perturbation theory already [5]. This implies that they cannot be understood as the exchange of energy carrying quanta, because such terms only appear in subleading order of perturbation theory.

In fact, we show that no transmission of energy from the sender to the receiver is necessary at all. An analysis of the energy cost of coupling a detector to the field shows that instead the energy the receiver requires to switch his detector depends on the sender's signal.

Our results do not only challenge our intuitive understanding of signalling in massless fields. Being leading order effects the described phenomena might soon be demonstrable in experiment, e.g., in cavity QED. The prospects of this we are currently investigating.

This is joint work with E. Martín-Martínez, and A. Kempf.

[1] Jonsson, R. H., E. Martín-Martínez, and A. Kempf. *In preparation*.

[2] Mann, R. B., and T. C. Ralph. Classical and Quantum Gravity 29.22 (2012): 220301. [3] Cliche, M., and A. Kempf. Phys. Rev. A 81, 012330 (2010).

[4] Cliche, M., and A. Kempf. Phys. Rev. D 83, 045019 (2011).

[5] Jonsson, R. H., E. Martín-Martínez, and A. Kempf. Phys. Rev. A 89, 022330 (2014)



### **Cosmological Perturbations in Antigravity**

Marius Oltean, *McGill University*

We compute the evolution of cosmological perturbations in a recently proposed Weyl-symmetric theory of two scalar fields with oppositely-signed conformal couplings to Einstein gravity. It is motivated from the minimal conformal extension of the Standard Model, such that one of these scalar fields is the Higgs while the other is a new particle, the dilaton, introduced to make the Higgs mass conformally symmetric. At the background level, the theory admits novel geodesically-complete cyclic cosmological solutions characterized by a brief period of repulsive gravity, or “antigravity,” during each successive transition from a Big Crunch to a Big Bang. We show that despite the necessarily wrong-signed kinetic term of the dilaton in the full action, its isotropic cosmological solutions are stable at the perturbative level.

### **Quantum control via relativistic probe atoms**

Chris Sutherland, *University of Waterloo*

We harness general relativistic effects to gain quantum control on a stationary qubit in an optical cavity by controlling the non-inertial motion of a different probe atom. Furthermore, we show that by considering relativistic trajectories of the probe, we enhance the fidelity of the quantum control.

### **An introduction to Information Theory and some of its applications: Black Hole Information Paradox and Renormalization Group Information Flow**

Fabio Grazioso, *Université de Montréal, Département d'Informatique et de Recherche Opérationnel, Laboratoire d'Informatique Théorique et Quantique*

Classical Information Theory (CIT) was devised to quantify the elusive concept of *information*. I introduce its basic definitions and some of its formal tools: the *information content* of a random variable, the *typical set* of an array of independent, identically distributed random variables, and some basic principles of *data compression*. I then describe the evolution of CIT into Quantum Information Theory (QIT), introducing concepts such as the *qubit*, the *pure* and *mixed states*, the *Holevo bound*, the *no-cloning theorem*, and the *quantum complementarity*, a concept that is exploited by some protocols of Quantum Cryptography. I then illustrate two examples of how the tools of QIT can be applied to other fields of physics.

First I describe the Black Hole (BH) Information Paradox, a theoretical problem related to the phenomenon of the Hawking radiation. Considering a BH starting in a pure state, after its complete evaporation only the Hawking radiation remains, which is shown to be in a mixed state with no correlation with the initial state. This either describes a non-unitary evolution of an isolated system, contradicting the evolution postulate of Quantum Mechanics, or it implies that the initial information content can escape the BH, therefore contradicting its description given by General Relativity. It can also be shown that the escape of information from a BH also violates the *no-cloning theorem*. I also describe some progress done toward the solution of the paradox.

As a second example I describe the Flow of Information along the Renormalization Group (RG). The RG is a tool developed at first to Quantum Field Theory, and then to other fields, such as Solid State Physics, Statistical Mechanics, and Cosmology. The c-theorem proves that a function exists, which is monotonically decreasing along the RG transformation, at least in the case of 2D systems. Because of this monotonicity, and the consequent irreversibility of the group transformation, some authors suggest an interpretation in terms of *information flow* for the c-function.

### **Relativistic enhancement of quantum optical metrology**

Aida Ahmadzadegan, *University of Western Ontario*

In this poster I will explore the applications of relativistic quantum information theory to metrology. By considering the relativistic effects on the transition probability of atoms moving through the optical cavities, we will characterize the perturbations of the general trajectories of an atom. Moreover, since the atom's transition rate depends on how the detector enters the cavity, we will show how this feature can be used as a test to measure the alignment of a cavity.

## **Dark Forces and Two Dark Matter Models: DDM and DDDM**

Satyen Baindur, *Canadian Nuclear Society*

The majority of Dark Matter candidates being considered today are electrically neutral (dark), non-relativistic (cold) Beyond Standard Model (BSM) theoretical proposals that result in single particles or a small set of particles with predicted lifetimes usually exceeding the age of the universe (i.e., „hyper-stable“ particles). Dynamical Dark Matter (DDM) [1], however, is a recently hypothesized form in which this last requirement is relaxed, by conjecturing instead a very large ensemble of particle species which might decay with finite lifetimes, but where the lifetimes are in a fine balance with their abundances, hypothesized as individually small. Any decays thus manage to preserve the overall dark matter density in the universe. Another Dark Matter Model, Double-Disk Dark Matter (DDDM) [2] similarly relaxes the minimality inherent in most Dark Matter models by hypothesizing a dark matter world that is just as rich and complex in particle species, their masses and their interactions, as the visible one. Separately, it has been suggested that Dark Matter could conceivably consist of particles that interact with each other via hypothesized Dark Forces [e.g., 3]. We consider the Sommerfeld enhancement of interaction cross sections in these contexts; astrophysical & cosmological consequences of these ideas, and their potential implications for detector experiments.

K. R. Dienes and B. Thomas, “Dynamical Dark Matter: I. Theoretical Overview,” arXiv:1106.4546.

J. J. Fan, A. Katz, L. Randall, and M. Reece, “Double-Disk Dark Matter”, arXiv:1303.1521

Nima Arkani-Hamed et al. “A Theory of Dark Matter”. arXiv:0810.0713

# Spin control in graphene quantum dots and graphene nanoribbon superlattices

Sanjay Prabhakar,<sup>1</sup> Roderick Melnik,<sup>1</sup> Shyam Badu,<sup>1</sup> Luis Bonilla,<sup>2</sup> and James Raynolds<sup>3</sup>  
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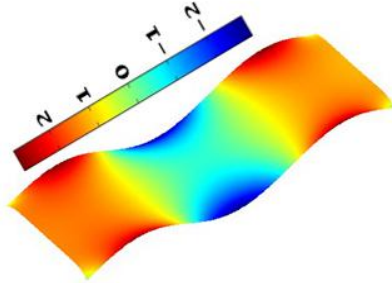
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In this poster session, we present the evolution of spin dynamics in graphene nanoribbon superlattices (GNSLs) with armchair and zigzag edges in the presence of a drift field. We determine the exact evolution operator and show that it exhibits spin echo phenomena due to rapid oscillations of the quantum states along the ribbon. The evolution of the spin polarization is accompanied by strong beating patterns. We also provide detailed analysis of the band structure of GNSLs with armchair and zigzag edges (Applied Physics Letters 103, 233112 (2013)).

We also provide the results associated to the in-plane oscillations of the relaxed shape graphene due to externally applied tensile edge stress along both the armchair and zigzag directions. Here thermo-electromechanical effects are treated via pseudomorphic vector potentials to analyze the influence of these coupled effects on the bandstructures of bilayer graphene quantum dots (QDs). We show that the total elastic energy density is enhanced with temperature for the case of applied tensile edge stress along the zigzag direction. We report that the level crossing between ground and first excited states in the localized edge states can be achieved with the accessible values of temperature. In particular, the level crossing point extends to higher temperatures with decreasing values of externally applied tensile edge stress along the armchair direction. Such kind of level crossing is absent in the states formed at the center of the graphene sheet due to the presence of three fold symmetry (arXiv:1401.4608) (2014).

Relaxed shape graphene



Graphene QDs formed by gate potential

