Communication Complexity of One-Shot Remote State Preparation

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Remote state preparation (RSP) is a variant of quantum teleportation introduced by Lo [4] in which Alice knows a classical description of the quantum state and wishes to help Bob prepare it, with the aid of shared entanglement and classical communication. In particular, Alice and Bob share entangled qubits, Alice is given the description of a state $Q(x)$, chosen from a subset of quantum states $\{Q(1), \cdots, Q(n)\}$, and their goal is to prepare that quantum state on Bob's side using an LOCC (Local Operations and Classical Communication) protocol. A relaxed version of this task is *approximate remote state preparation* in which the goal is to prepare an approximation σ_x of a quantum state $Q(x)$. We define the error of a protocol for approximate remote state preparation in terms of the fidelity between $Q(x)$ and σ_x. We say a protocol prepares a state with *worst-case error* at most ϵ, if for every $x \in \{1, \cdots, n\}$, $\mathrm{F}(Q(x), \sigma_x) \geq \sqrt{1-\epsilon^2}$. Similarly, a protocol has *average-case error* at most ϵ with respect to a probability distribution p, if $\sum_{x=1}^{n} p_x \mathrm{F}(Q(x), \sigma_x) \geq \sqrt{1-\epsilon^2}$.

In [4], Lo gave several examples of ensembles which can be remotely prepared using a one-way protocol with classical communication cost less than quantum teleportation. However, he conjectured that to prepare an arbitrary pure state remotely, one needs to send classical bits in the same asymptotic rate as in quantum teleportation i.e., 2 classical bits per qubit. In [1], it was shown that the asymptotic classical communication cost of remote state preparation of a general state ranges from one bit per qubit in the high entanglement limit to infinite bit per qubit in the case of no shared entanglement. Later in [2], Bennett *et al.* showed that approximate remote state preparation with small worst-case error ϵ requires asymptotic rate of one bit of classical communication per qubit from Alice to Bob. They also showed that this amount of classical communication is sufficient. In [3], Jain studied remote state preparation in the *one-shot* scenario. He considered the total communication cost instead of the rate of communication in the case that there is no restriction on the amount of entanglement. He showed that the communication cost required for *exact* remote state preparation is at least $\mathsf{T}(Q)/2$ and approximate RSP with worst-case error at most ϵ can be solved with communication at most $\frac{8}{\left(1-\sqrt{1-\epsilon^2}\right)^2}(4\mathsf{T}(Q)+7)$, where $\mathsf{T}(Q)$ denotes maximum possible mutual information in an encoding Q. It is rather unsatisfactory that the upper bound for the approximation version be *larger* than the lower bound for the *exact* version of the problem. We would expect the complexity of the problem to decrease as the error in approximation increases.

In this work, we give a tighter characterization of the communication complexity of *one-shot* remote state preparation in two different cases. First, we consider RSP with average-case error at most ϵ, and show that its communication complexity is bounded from below and above by the notion of smooth max-information Bob has about Alice's input. Then, we consider RSP with worst-case error at most ϵ, and give lower and upper bounds for its communication complexity in terms of smooth max-relative entropy and show that our upper bound is a factor of $\log\log N$ tighter than that of [3]. Also, we show that the communication cost may reduce dramatically by allowing more error and considering average-case error instead of worst-case error. In particular, we show that for every $\epsilon \in [0, \frac{1}{\sqrt{2}})$, there exist a set of quantum states of dimension N and a probability distribution p_ϵ over this set for which there is a $\log N$ gap between the worst-case error and average-case error remote preparation of that set. In addition, we exhibit a set of quantum states, which illustrates the heavy dependence of the the gap between the worst-case error and average-case error communication complexity on the error parameter ϵ. This example also shows that the more skewed the probability distribution is, the bigger the gap between worst-case and average-case error can be.

References

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