THE SUPERSYMMETRIC RUIJSENAARS-SCHNEIDER MODEL AND ITS INTEGRABILITY

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The Calogero-Sutherland (CS) model describes a quantum mechanical system of N particles interacting pairwise in 1 dimension. Its one-parameter deformation, called the (trigonometric) Ruijsenaars-Schneider (RS) model, is (often) considered as its relativistic extension since the algebra satisfied by its hamiltonian, momentum and boost operators, is the Poincaré algebra. Moreover, in the limit $c \to \infty$ (where c is playing the role of speed of light), it reduces to the CS model. These two models are integrable because there are $2N$ mutually commuting quantities whose set includes the hamiltonian. It is also a remarkable result that the eigenfunctions of these models are the Jack and Macdonald symmetric polynomials (in the CS and RS cases respectively).

Supersymmetric generalization of quantum mechanical models which preserves integrability is an important field of research in mathematical physics. For the case of CS model, this has been formulated in 1993. Its eigenfunctions (obtained in early 2000) are superpolynomials that depend upon two families of variables – commuting and anticommuting – and which remain invariant when both type of variables are simultaneously permuted. These are naturally called the Jack superpolynomials. Furthermore, it turns out that they can also be defined purely by combinatorial methods and their study opened up the field of symmetric functions in superspace.

On the other hand, the quest for an integrable one-parameter deformation of the supersymmetric CS model is a long standing problem. The difficulty lies in that standard supersymmetrization methods no longer work in case of the RS model. Very recently, such an extension has been discovered and its eigenfunctions are the superspace analogue of Macdonald polynomials which have been constructed in the last two years by combinatorial techniques. The proof of integrability of this model makes use of the Hecke algebra and the related Cherednik operators.

In this talk, I will present the supersymmetric formulation of the trigonometric RS model. Its underlying algebra, that is, the (anti)commutation relations satisfied by the hamiltonian, momentum, boost and supercharges, is now the Poincaré superalgebra. Then, I will present its eigenfunctions, the Macdonald superpolynomials, with some of their (amazing) combinatorial properties. Finally, I will show how the commuting conserved quantities can be constructed with the aid of the Cherednik operators. These original results are based on the following references.

References

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