Charge Quantization from a Number Operator

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Background. It is known that electric charge quantization arises in the presence of Dirac monopoles, and in the context of grand unified theories. More recently, Hellerman, Kehayias, and Yanagida have proposed that charge quantization can come about in non-linear sigma models, [1].

Here, we reveal a striking new resolution to the question, *Why is electric charge quantized?*. Perhaps electric charge is quantized because it happens to be given by a number operator.

To this end, we consider the possibility of a more abstract mathematical structure behind the Lie group representations of the standard model. One option for this structure is that of the normed division algebras over $\mathbb{R}$. Only four of these algebras exist, and three of them, namely the reals, the complex numbers and the quaternions (sigma matrices), are already central to particle physics. Here, we explore the role of the fourth division algebra, the octonions.

In earlier years, [2], Günaydin and Gürsey showed $SU_c(3)$ quark structure in the split octonions. Later, in [3], they showed anti-commuting ladder operators, α_i, within that model. Our new results are consistent with the chromodynamic quark model of [2] and [3].

New results. We show a direct route, via ladder operators, to a new $U(1)$ generator. This $U(1)$ generator behaves like electric charge, thereby allowing us to identify not only the quark states, but also new states behaving like the electron and neutrino. Our proposed electric charge turns out to be proportional to a number operator, consequently illuminating why it is quantized.

$$Q \equiv \frac{1}{3}\sum_{i=1}^{3}\alpha_i^{\dagger}\alpha_i = \frac{N}{3}. \tag{1}$$

Using only a trio of ladder operators, α_i, and their conjugates, we construct a pair of *minimal left ideals,* which is shown to transform under $SU_c(3)$ and $U_{em}(1)$ as does a full generation of the standard model.

The first of the minimal left ideals, S^u, behaves under electric charge, Q, and $su_c(3)$, Λ, according to the following table.

Q	Λ	S^u	ID
0	1	$\omega\omega^\dagger$	ν
1/3	$\bar{3}$	$\alpha_i^\dagger\omega\omega^\dagger$	$\bar{d}_i$
2/3	3	$\alpha_i^\dagger\alpha_j^\dagger\omega\omega^\dagger$	u_k
1	1	$\omega^\dagger$	e^+

(2)

Here, objects $\omega\omega^\dagger$, $\alpha_i^\dagger\omega\omega^\dagger$, ... are simply basis vectors of S^u. The object ω is defined as $\omega \equiv \alpha_1\alpha_2\alpha_3$.

The complex conjugate of S^u gives a new linearly independent ideal, S^d, in the space of octonionic maps. In this new formalism, each particle and anti-particle pair are related by *only* the complex conjugate, $i \mapsto -i$. These two minimal left ideals, or generalized spinors, S^u and S^d, are depicted in the figure below.

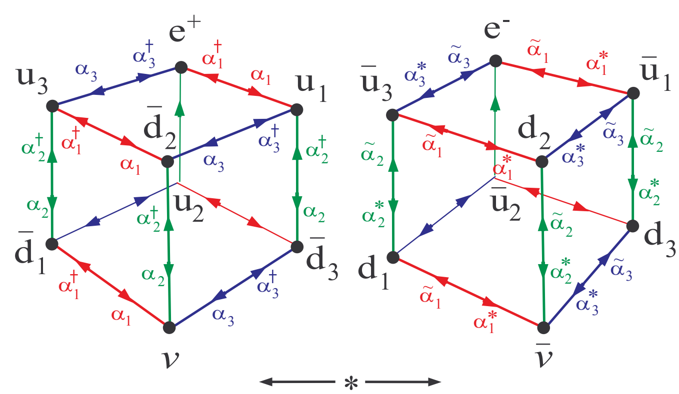

FIG. 1. A full generation represented by cubes S^u (left) and S^d (right). Quark and electron states may be viewed as excitations from the neutrino or anti-neutrino.

[1] S. Hellerman, J. Kehayias, T. Yanagida, Physics Letters B 731C (2014), pp. 148-153, arXiv:1312.6889 [hep-th]
[2] M. Günaydin, F. Gürsey, J. Math. Phys., Vol. 14, No. 11 (1973)
[3] M. Günaydin, F. Gürsey, PRD, Vol. 9, No. 12 (1974)