Near Horizon Geometries and Black Hole Thermodynamics: a Lesson in Limits

It is well known that the spacetime in the vicinity of an extremal black hole horizon has an enhanced symmetry group. This enhancement of symmetry is due to the attractor mechanism, whereby the entropy of extremal black holes cannot depend on any moduli of the theory. The near horizon geometry (NHG) in D dimensions is typically a product (either direct or twisted) of AdS_2 and S^{D-2}, which has allowed for an interpretation of extremal black hole entropy in terms of microscopic states in a dual CFT. However, the standard limiting procedure for finding the NHG obscures the connection between the NHG and the bulk space-time: in particular, it is not immediately clear what the horizon maps to in the NHG and which Killing symmetries in the NHG are present in the bulk spacetime and which are not. The standard limiting procedure further obscures the meaning of the NHG: I will show that the NHG is to be thought of as a tangent space to the horizon, with the degenerate horizon mapping to a degenerate null hypersurface in the NHG. This distinction becomes extremely important when considering the extremal limit of Schwarzschild de Sitter. Standard lore would have one believe that the volume between the coalescing horizons remains finite in the extremal limit, leading to the Nariai spacetime whose two cosmological horizons are interpreted as the two limiting horizons. Importantly, this suggests that extremal Schwarzschild de Sitter is at finite temperature. However, I will show that this interpretation is problematic; the Nariai spacetime should really be thought of as the NHG of the extremal Schwarzschild de Sitter spacetime. When viewed this way, I will show that extremal Schwarzschild de Sitter is indeed at zero temperature, as an extremal horizon should be.